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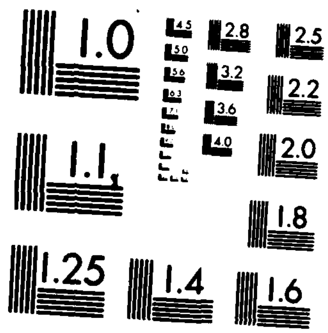

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SUBOPTIMAL THRESHOLD DETECTION IN NARROWBAND NON-GAUSSIAN NOISE

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ABSTRACT

The Middleton Class A narrowband non-Gaussian noise model is considered. The performance of various simple suboptimum threshold detectors is examined and compared with the performance of the optimum detector. For example, it is shown that for most cases of interest the blanker has nearly optimal performance while the soft limiter and hard limiter have substantially less than optimal performance. Then, a new approximation to the locally optimum detector nonlinearity is developed using ideas from the theory of robust detection.

I. INTRODUCTION

Various attempts have been made to develop models of non-Gaussian noises (see, for example, [1-13]). Among the most general models are those developed by Middleton [7-12]. Middleton divides non-Gaussian noise into two classes, A and B. (There has also been consideration of a Class C which contains noises which are sums of Class A and Class B components [9].) Class B noises are broadband, i.e. those with spectra broader than the passband of the receiver. Class A noises are narrowband, i.e. have spectra comparable to or narrower than the receiver passband.

In a previous paper [1] we examined the Middleton Class A noise model. We showed that, in a wide variety of cases, the first-order noise probability density function (PDF), which is an infinite weighted sum of Gaussian PDF's, can be closely approximated by the (normalized) sum of the first two terms in the series. We also showed that the detector which is locally optimum for this approximation performs very well for the original Class A model.

In this paper, after a brief review of previous work in Section II, we examine the performance of other even simpler suboptimum threshold detectors. We show that in almost every case at least one of these very simple detectors is nearly optimal. In the Section IV we return to the two term approximation discussed above. Anticipating that this simple approximation would not work in every case we developed a somewhat more sophisticated scheme involving ideas from robust detection. This scheme is discussed in Section IV.

II. PREVIOUS WORK

In order to formulate his model, Middleton [7-12] assumes the noise has the form $X(t)+N(t)$ where $N(t)$ is a Gaussian background component and

$$X(t)=\sum_j U_j(t,\vartheta) \qquad (1)$$

where U_j denotes the j^{th} received waveform from an interference source and ϑ is a random parameter. He then assumes that the waveform receptions are Poisson distributed in time and shows that the normalized (to unit variance) noise density $f(x)$ can be approximated canonically by

$$f(x)=\sum_{m=0}^{\infty} K_m f(x;\sigma_m^2) \qquad (2)$$

where $f(x;\sigma^2)$ is the zero-mean Gaussian PDF with variance σ^2. The variance σ_m^2 of the m^{th} density is given by

$$\sigma_m^2=\frac{m/A+\Gamma}{1+\Gamma} \qquad (3)$$

and the coefficient K_m is given by

$$K_m=e^{-A}\frac{A^m}{m!} \qquad (4)$$

where A and Γ are the two basic parameters of the model. The first parameter, A, is called the "overlap index" and is defined by $A=\nu T$, where ν is the rate of the homogeneous Poisson process which governs the generation of the interfering waveforms U_j and T is the mean duration of a typical interfering signal. The other parameter, Γ, is given by the ratio of the power in the Gaussian portion of the interference to the power in the Poisson component.

Middleton has shown that, by adjusting the parameters A and Γ, the density f given in (2) can be made to fit a great variety of non-Gaussian noise densities [9-12]. Also, the parameters A and Γ are physically motivated and can be directly estimated (see [12,9]). Unfortunately the model (2) is cumbersome. For example, in [11], Spaulding and Middleton exhibit the optimal nonlinearity for detection (i.e. the likelihood ratio $f(x-s_1)/f(x-s_0)$) and point out that this detector structure is likely to be computationally burdensome and uneconomical. Thus we would like to develop detector nonlinearities having simpler structure but which retain the desirable properties of the one given in [11].

In [1] we considered the (normalized) M-term truncation of the Class A noise PDF given in (2), i.e.

$$f^{(M)}(x)=\frac{\sum_{m=0}^{M-1} K_m f(x;\sigma_m^2)}{\sum_{m=0}^{M-1} K_m} \qquad (5)$$

We showed that $f^{(M)}$ is a very good approximation to f for small values of M. In particular, for the problem of threshold (or locally optimum) detection (i.e. small signal, large time-bandwidth product), the detector designed assuming the noise has density $f^{(M)}$ performs virtually as well as the optimal detector when the noise is actually given by the full Class A model f, even for

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values of M as small as 2.

In the next section we will examine the performance of several suboptimum detectors whose structure is even simpler than the ones considered in [1], but first we will give a brief summary of the necessary results from the theory of locally optimum detection. The details of this theory may be found in many places [14-17,11].

Under mild regularity conditions the (asymptotic) performance (or processing gain) achievable using a given detector nonlinearity $g(x)$ when the (independent identically distributed) noise process has first order PDF $h(x)$ is given by the efficacy functional

$$\eta(g,h) = \frac{\left[\int_{-\infty}^{\infty} g(x)h'(x)dx\right]^2}{\int_{-\infty}^{\infty} g^2(x)h(x)dx} \tag{6}$$

For a given noise PDF $h(x)$ the locally optimum detector nonlinearity $g_h^*(x)$ is the solution to

$$\eta^*(h) = \max_{g} \eta(g,h) \tag{7}$$

and has the form

$$g_h^*(x) = \frac{-h'(x)}{h(x)} \tag{8}$$

The performance of this locally optimum nonlinearity is given by

$$\eta^*(h) = \eta(g_h^*,h) = \int_{-\infty}^{\infty}\left[\frac{h'(x)}{h(x)}\right]^2 h(x)dx. \tag{9}$$

The functional $\eta^*(h)$ is also known as Fisher's Information for h. An interesting (and well known) fact about the function $\eta^*(h)$ is that it is minimized over all PDF's by the Gaussian PDF. In fact, the locally optimum detector nonlinearity for Gaussian noise is the linear detector ($g(x)=x/\sigma^2$) which has performance equal to unity for all noise PDF's.

The Class A noise PDF f in (2) is highly non-Gaussian, and it is often the case that $\eta^*(f)>>1$. In [1] it was shown that for very small values of M, $f^{(M)}$ and $g_M^*(x)$ closely approximate f and its locally optimum nonlinearity $g_f^*(x)$. Moreover, for a wide range of values of the parameters) we saw that the processing gain achievable using g_2^* is extremely close to that achievable using g_f^*. That is, $\eta(g_2^*,f) \approx \eta^*(f)$. Of course, there are situations where even simpler detector nonlinearities are desirable. In the next section we examine the performance of three commonly-used very simple suboptimum nonlinearities.

III. THREE VERY SIMPLE SUBOPTIMUM NONLINEARITIES

In Figure 1 the locally optimum nonlinearity for a Class A noise PDF is plotted for $x \geq 0$ (since f is symmetric, g_f^* is antisymmetric, i.e. $g_f^*(-x) = -g_f^*(x)$). The parameters (A=0.35, Γ=0.0005) of the Class A model used in Figure 1 are used in [9-12] to fit "interference (probably) from nearby powerline, produced by some kind of equipment fed by line" [9]. The vertical dashed lines at x = 0.06 and x = 0.10 divide the x-axis into three regions $S_1 = \{|x| \leq 0.06\}$, $S_2 = \{0.06<|x|\leq 0.10\}$, and $S_3 = \{|x|>0.10\}$ which are the regions where $g_f^*(x)$ is approximately linear (S_1), returning to zero (S_2), and approximately zero (S_3). Evaluating the probability under f of each of these regions (or, more intuitively the fraction of the data we should expect to fall in each region) we have that $Pr(S_1) \approx 0.71$, $Pr(S_2) \approx 0.01$, and $Pr(S_3) \approx 0.28$. Thus we see that all but about 1% of the time the data will fall in the approximately linear region or the approximately zero region. This leads us to believe that g_f^* can be closely approximated by a blanker g_B^c (also called a hole puncher) which is shown in Figure 2a. For comparison we have also examined the performance of a soft limiter g_{SL}^c (or clipper) and a hard limiter g_{HL}^c (or sign detector) which are shown in Figures 2b and 2c, respectively.

In Table 1 we have given the processing gain achievable using the locally optimum nonlinearity g_f^*, the blanker g_B^*, the soft limiter g_{SL}^*, and the hard limiter g_{HL} (Note that the stars on g_B and g_{SL} indicate that the optimal value of c is used). We have included each of the examples used in [1] as well as two others. In each case we see that the blanker is nearly optimal while the soft limiter and the hard limiter have substantially less than optimal performance. The one exception to this is the last example (A=1.0, Γ=0.1) where the soft limiter is nearly optimal and even the hard limiter outperforms the blanker. Not surprisingly the locally optimum nonlinearity for this case (shown in Figure 3) is more closely approximated by a soft limiter than a blanker. We must stress though that, based on our experience, this seems to be an unusual case. In fact Table 1 is quite representative of our findings in general.

Another issue of importance when considering various detector nonlinearities is that of robustness or sensitivity. Since the blanker and soft limiter each only depend on one parameter (the "cut-off" parameter c), it is fairly straightforward to examine their robustness. (Note that a hard limiter does not depend on such a parameter.) In Figure 4 we plotted the processing gain achievable using g_B^c and g_{SL}^c versus the cut-off parameter c for the example (A=0.35 Γ=0.0005) considered in Figure 1. It would seem from the smoothness and flatness of the curves in Figure 4 near their respective maxima that both the soft limiter and the blanker are quite insensitive to variations in the cut-off parameter.

On the other hand, in Figure 5, the same two curves are plotted using a different scale on the abscissa. This new scale is not the cut-off parameter c but the probability of the set $\{-c \leq x \leq c\}$ under the Class A PDF $f(x)$. As mentioned above, this can be thought of as the fraction of the data which we can expect to fall in the linear region of the detector nonlinearity (cf. S_1 in the first paragraph of this section). Since any estimate of c^*, the optimal value of the cut-off parameter, would presumably come from some version of an empirical PDF (see [12]), Figure 5 is likely to be a more reasonable way than Figure 4 to examine the sensitivity of the blanker and soft limiter to uncertainties in estimating c^*. Results similar to Figure 5 have also been obtained for the Middleton Class B (broadband) noise model by Ingram and Houle [18].

It is not unreasonable at first thought to assume that this change in scale would cause little change in the relative smoothness and flatness of the two curves. In fact Figure 5 shows quite strikingly that the blanker is very sensitive while the soft limiter is very insensitive near their respective maxima. This example is again quite representative of a wide variety of other cases.

IV. A ROBUST APPROXIMATION SCHEME

In this section we present a new scheme for detection in noise which is reasonably well-modeled by the Class A density (2). This scheme requires some background on the theory of asymptotically robust detection. This theory was first developed in a paper by Martin and Schwartz [19] for the case of nearly Gaussian noise. It was then extended by Kassam and Thomas [20] to mixture models with general "nominal" noise densities. We now present the needed results from [20].

In [20] an ε-mixture (or ε-contaminated) class of PDF's of the form

$$\mathbf{F} = \{f \mid f(x) = (1-\varepsilon)h_0(x)+\varepsilon h_1(x),\ h_1 \in \mathbf{H}\} \quad (10)$$

was considered, where h_0 is a fixed element of H satisfying certain regularity conditions and H is the class of all bounded symmetric PDF's. They showed that the detector nonlinearity which has the best lower bound over the class F on asymptotic performance, i.e., the minimax locally optimum detector nonlinearity $g_{h_0}^R$, has the form

$$g_{h_0}^R(x) = \begin{cases} \dfrac{-h_0'(-c)}{h_0(-c)} & x<-c \\ \dfrac{-h_0'(x)}{h_0(x)} & |x| \le c \\ \dfrac{-h_0'(c)}{h_0(c)} & x>c \end{cases} \quad (11)$$

where c is chosen to satisfy

$$\int_{-c}^{c} h_0(x)dx - \frac{2h_0^2(c)}{h_0'(c)} = \frac{1}{1-\varepsilon} \quad (12)$$

As we see from (11), the robust detector nonlinearity is equal to the locally optimal nonlinearity for the nominal PDF h_0 (see (6)) for $|x|$ less than the censoring constant c, and it is constant for $|x|$ greater than c, i.e., $g_{h_0}^R$ is a censored version of $g_{h_0}^*$.

In [1] we saw that in most cases the locally optimum detector nonlinearity g_f^* for the Class A PDF (2) can be closely approximated by the locally optimum nonlinearity g_2^* for the PDF containing just the first two terms (i.e., $f^{(2)}$ as in (5)). In the cases where there was any substantial difference between the two nonlinearities it was in the tails. Beyond 4 or 5 standard deviations, the two term nonlinearity g_2^* continued its linear path while the infinite term nonlinearity flattened virtually to the horizontal. In Figure 6 these two nonlinearities have been plotted.

The horizontal (dotted) line added to Figure 6 corresponds to the robust detector nonlinearity for the class F with $h_0 = f^{(2)}$ and $\varepsilon = \sum_{m=2}^{\infty} K_m$. That is, instead of simply discarding all the terms in the Class A PDF beyond the first two as in [1], we have "robustified" with respect to this remainder. It is clear from Figure 6 that this robustified two term nonlinearity g_2^R may often be an even better approximation to g_f^* than g_2^*. Our conviction has been that, in those cases where the two term locally optimum nonlinearity g_2^* does not perform in a nearly optimal fashion (assuming the noise actually has the Class A density f as in (2)), the robust version g_2^R will. However, as of this writing we have been unable to find a single case where g_2^* does not perform virtually as well as g_f^*.

Still there are justifications for this robust scheme. First, in every case we have considered the robust version performs about as well or even better than the two term optimal version. Even though these differences are negligible they do not contradict the conviction stated above. Of course, this requires further exploration. Second, it has been shown by Berry [13] that the Class A model would generaly be useful when the noise is dominated by two components: receiver noise and one strong interferer. If this is the case, the Class A model may be chosen solely on the basis of its fit for the first two (dominating) terms and the remaining terms may not really be accurate. Clearly, due to its minimax design the robust version will be less sensitive to these inaccuracies than the two term optimal detector. In fact, it might even outperform the "optimal" nonlinearity g_f^* if the true noise has other small interferers which are not at the appropriate distances and powers necessary to be accurately described by the Class A model beyond the first two terms.

V. SUMMARY

In this paper we have examined the performance of some simple suboptimum detector nonlinearities. For most Class A noises the blanker has nearly optimal performance while the soft limiter and hard limiter have significantly lower performance. On the other hand the performance of the blanker seems to be far more sensitive to errors in estimating the optimal cut-off parameter.

In [1] we showed that the Middleton Class A noise model can often be approximated closely by the (normalized) sum of just the first few terms. In fact, in many cases, two terms are sufficient. This was especially clear when we looked at the efficacy of the detector nonlinearity which is locally optimum for two terms of the Class A model and found it comparable to the efficacy of the full locally optimum detector. In this paper we also developed a robust scheme which needs some further examination but which we feel offers a number of advantages over the methods of [1].

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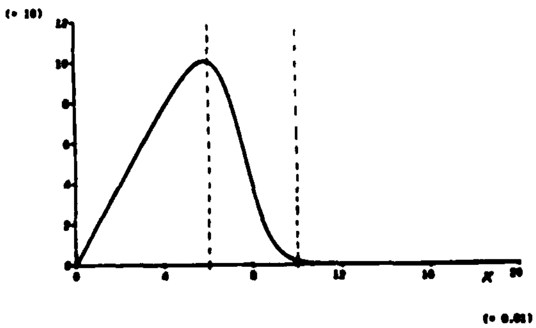

Figure 1. Locally optimum detector nonlinearity for Class A noise PDF f (linear scales) (A=0.35, Γ=0.0005)

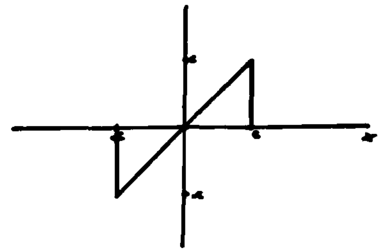

Figure 2a. The blanker (hole puncher).

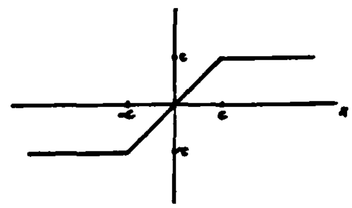

Figure 2b. The soft limiter (clipper).

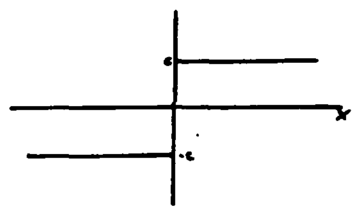

Figure 2c. The hard limiter (sign detector).

A,Γ	0.35, 0.0005	0.0001, 50	0.1, 0.001	0.1, 0.1	1.0, 0.0001	1.0, 0.1
g^*	1340	1.02	892.6	9.2	3299	2.39
g^*_B	1325	1.02	890.2	9.0	3221	1.89
g^*_{SL}	730	1.02	885.1	7.6	816	2.20
g^*_{HL}	839	0.65	816.7	5.9	886	1.89

Table 1. Processing gain (efficacy) achievable using optimal (g^*), blanker (g^*_B), soft limiter (g^*_{SL}), and hard limiter (g^*_{HL}) nonlinearities.

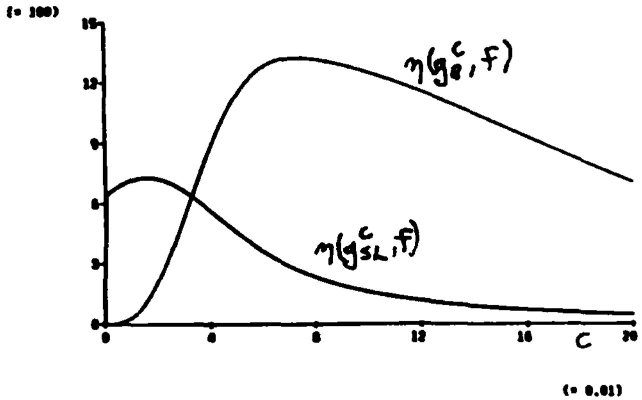

Figure 4. Efficacy (processing gain) of soft limiter g_{SL}^c and blanker g_B^c (A=0.35, Γ=0.0005).

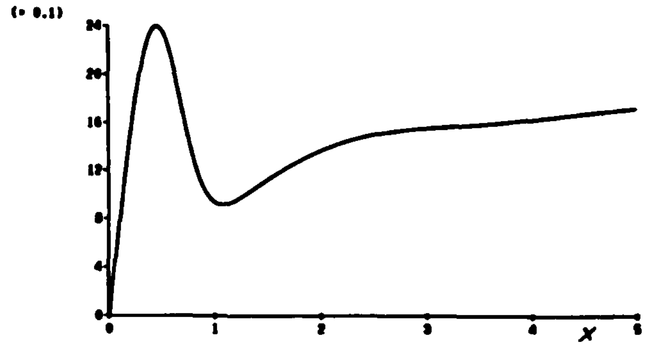

Figure 3. Locally optimum detector nonlinearity for Class A noise PDF f (linear scales) (A=1.0, Γ=0.1).

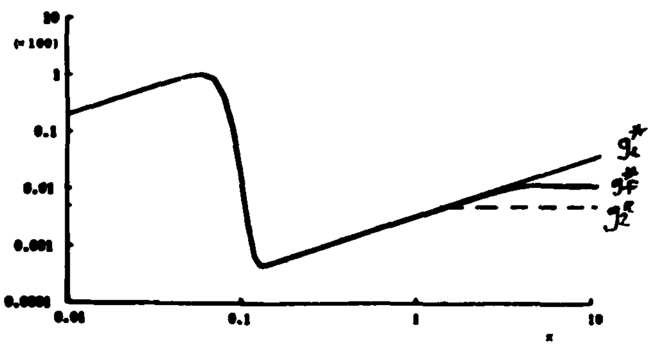

Figure 6. Locally optimum detector nonlinearities g_L^* and g_F^* and robust nonlinearity g_2^R (dashed line) (A=0.35, Γ=0.0005).

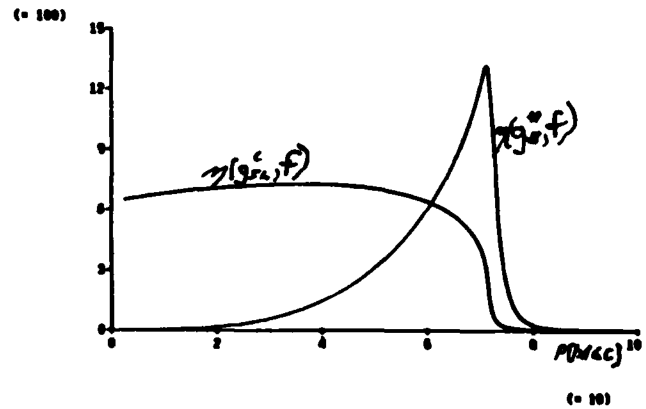

Figure 5. Efficacy (processing gain) of soft limiter g_{SL}^c and blanker g_B^c (A=0.35, Γ=0.0005). Abscissa scale different from Fig. 4.

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